Structure-based control of complex networks with nonlinear dynamics

用非线性动力学对复杂网络进行结构控制

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如何仅仅通过潜在的网络结构来控制一个系统

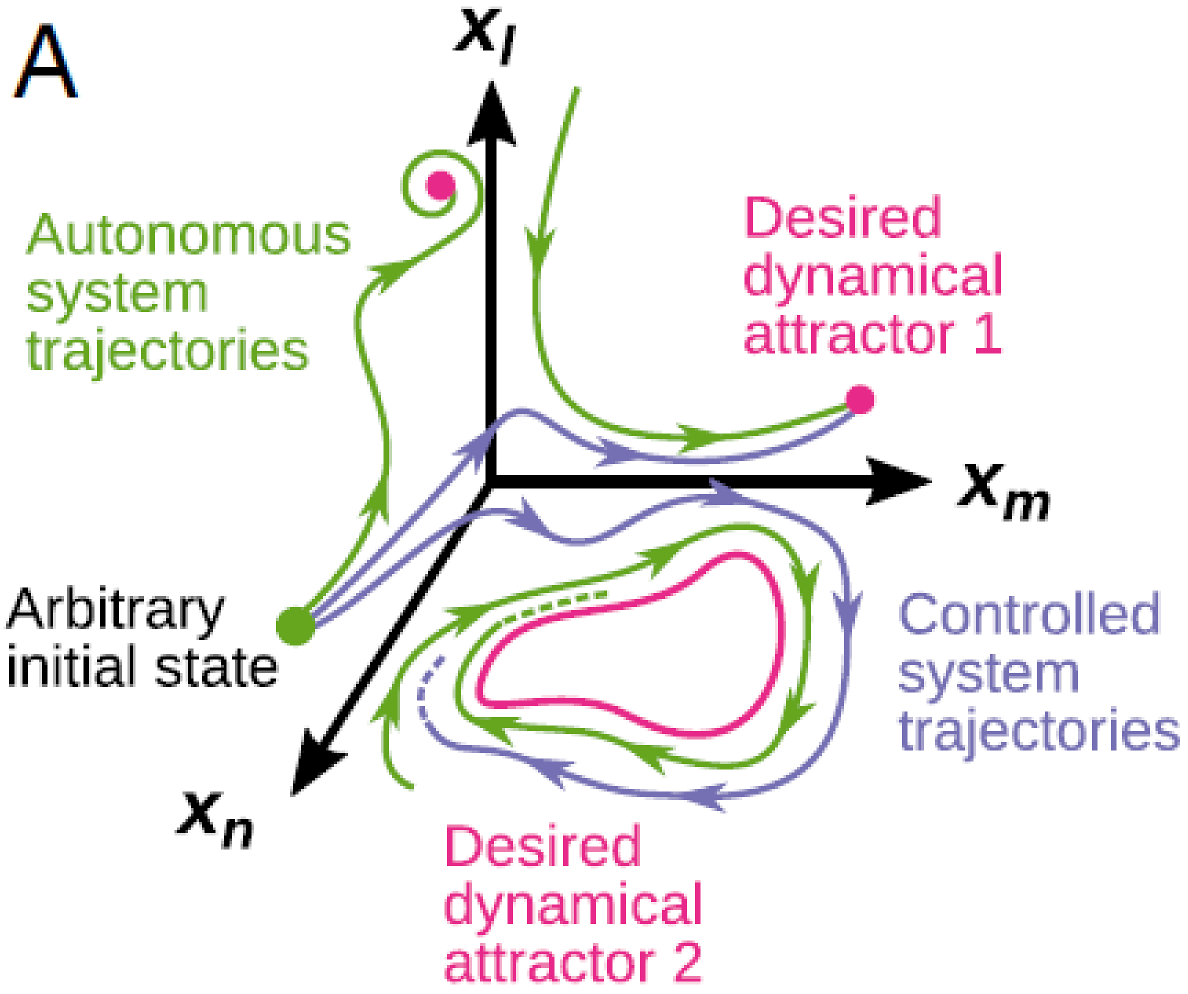
- How to control a complex network system such as biological, technological, and social systems over which nonlinear dynamical processes take place. Which network elements need to be controlled, and through which control actions, to drive the system toward a desired control objective.

结构可控理论——控制复杂网络的一般方法

- Structural controllability (SC), which assumes unspecified linear dynamics or linearized nonlinear dynamics, allows the identification of the minimal number of nodes whose receiving an external signal u(t) drives the system into a state of interest.
- Despite its success and widespread application, SC may give an incomplete view of the network control properties of a system.

新的方法——反馈节点集控制理论

- Feedback vertex set control (FC),is a structure-based control method that can make conclusions about the long term dynamics of a system using solely the network structure.
- FC is a mathematical formalization of the following idea: To drive the state of a network to any one of its naturally occurring end states (dynamical attractors), one needs to manipulate a set of nodes that intersects every feedback loop in the network–the feedback vertex set(FVS).
- FC does not use a controller or driver signal, and instead considers node state override as its control action.



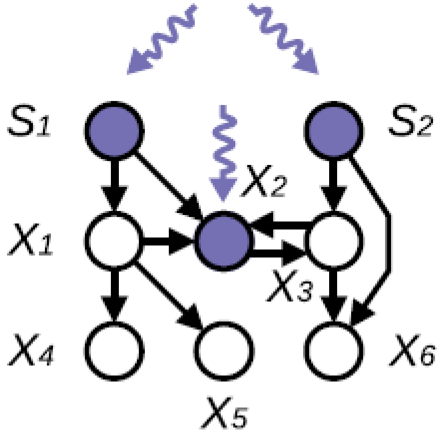
Control of the source nodes and of the FVS of a network guarantees that we can guide it from any initial state to any of its dynamical attractors regardless of the specific form of the functions.

we refer to this attractor-based control method as FC, and to the group of nodes that need to be manipulated by FC as an FC node set.

非线性动力学对复杂网络结构的控制

$$dX_i/dt = F_i(X_i, X_{I_i}, t), \quad [1]$$

$$dS_j/dt = G_j(t), \quad [2]$$

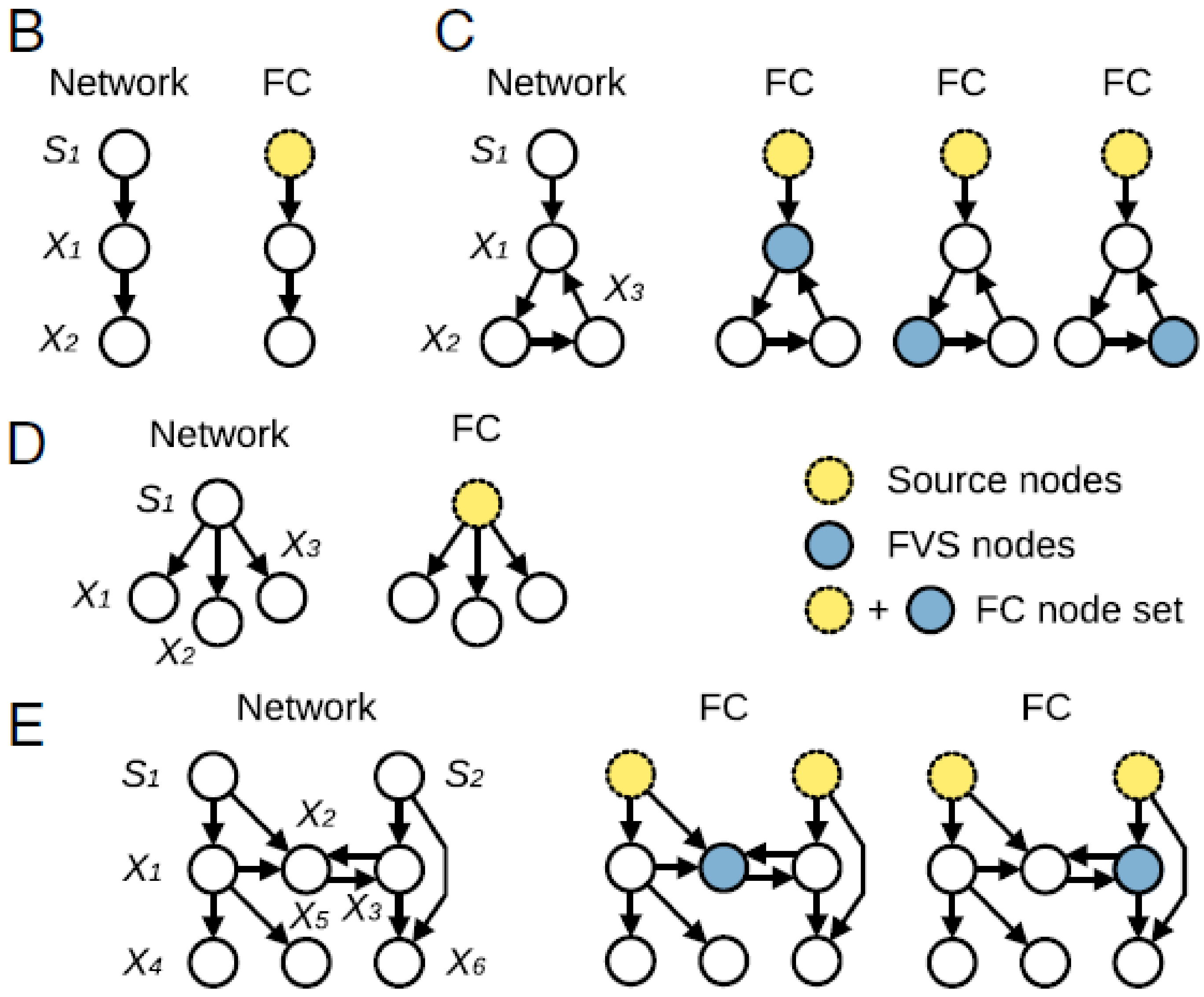


$S_j(t)$：source node variables（源节点变量）；$X_i(t)$：internal node variables（内部节点变量）

N：节点总数；t：时间；N_s：源节点的总数

$i = 1,\ldots,N-N_s$； $j = N-N_s+1,\ldots,N$

I_i：predecessor nodes（前驱节点）



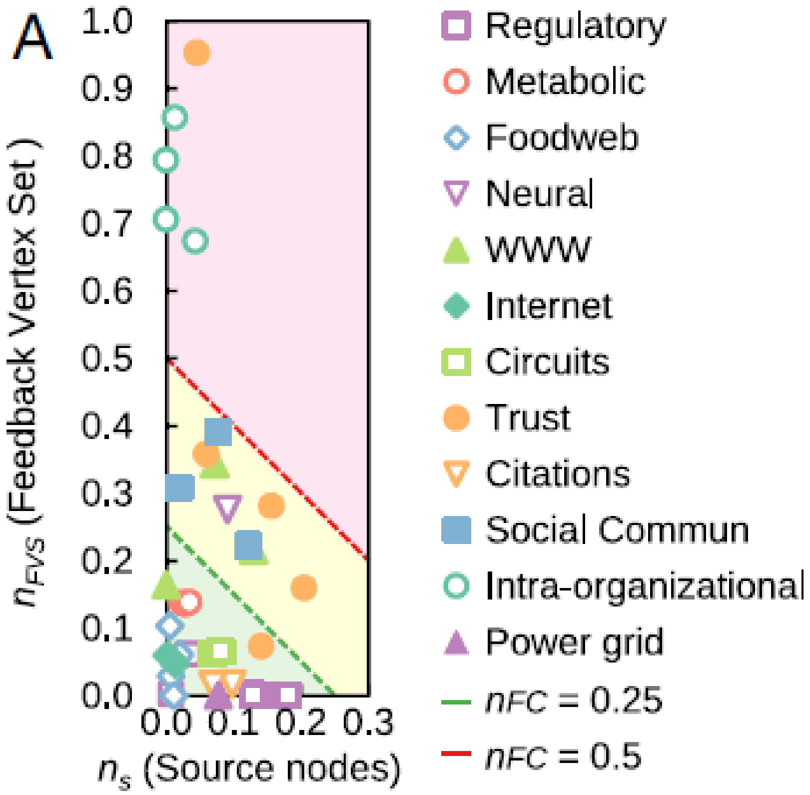
FC in simple networks

FC requires control of the source nodes (yellow nodes with dotted outlines) and of all cycles by control of the FVS (blue nodes with solid outlines).

For C, a source node connected to a cycle, FC requires controlling the source node S1 and any node Xi in the cycle, the FVS in this network.

D consists of a source node with three successor nodes, and FC requires controlling only the source node S1, because there are no cycles in the network.

将反馈节点集控制的理论应用于真实的网络



N_{FC}：反馈节点集控制理论需要控制的最少节点

N：节点的总数

$n_{FC}=N_{FC}/N$：FC所占的比率，用来评估控制一个网络的困难程度

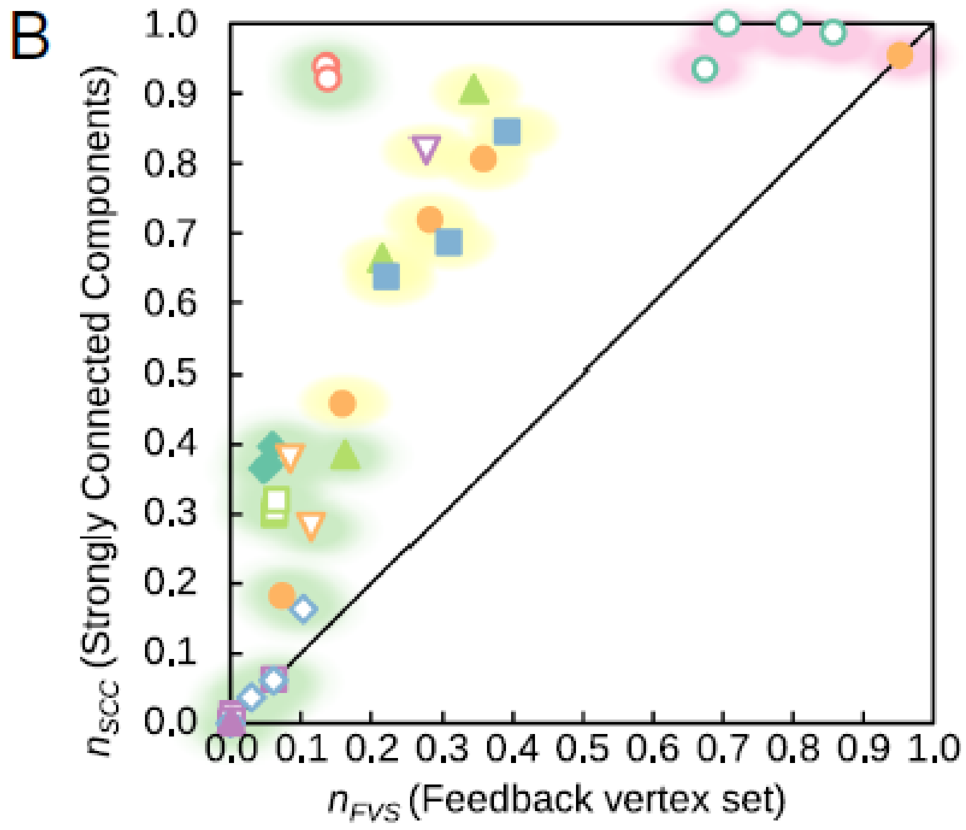
n_{FVS}：反馈节点集所占的比率

n_S：源节点所占的比率

$n_{FC} = n_{FVS} + n_S$

Most types of biological networks (gene regulatory, metabolic, and food web networks) require control of a smaller fraction of nodes than social networks (trust, social communication, and intraorganizational networks)

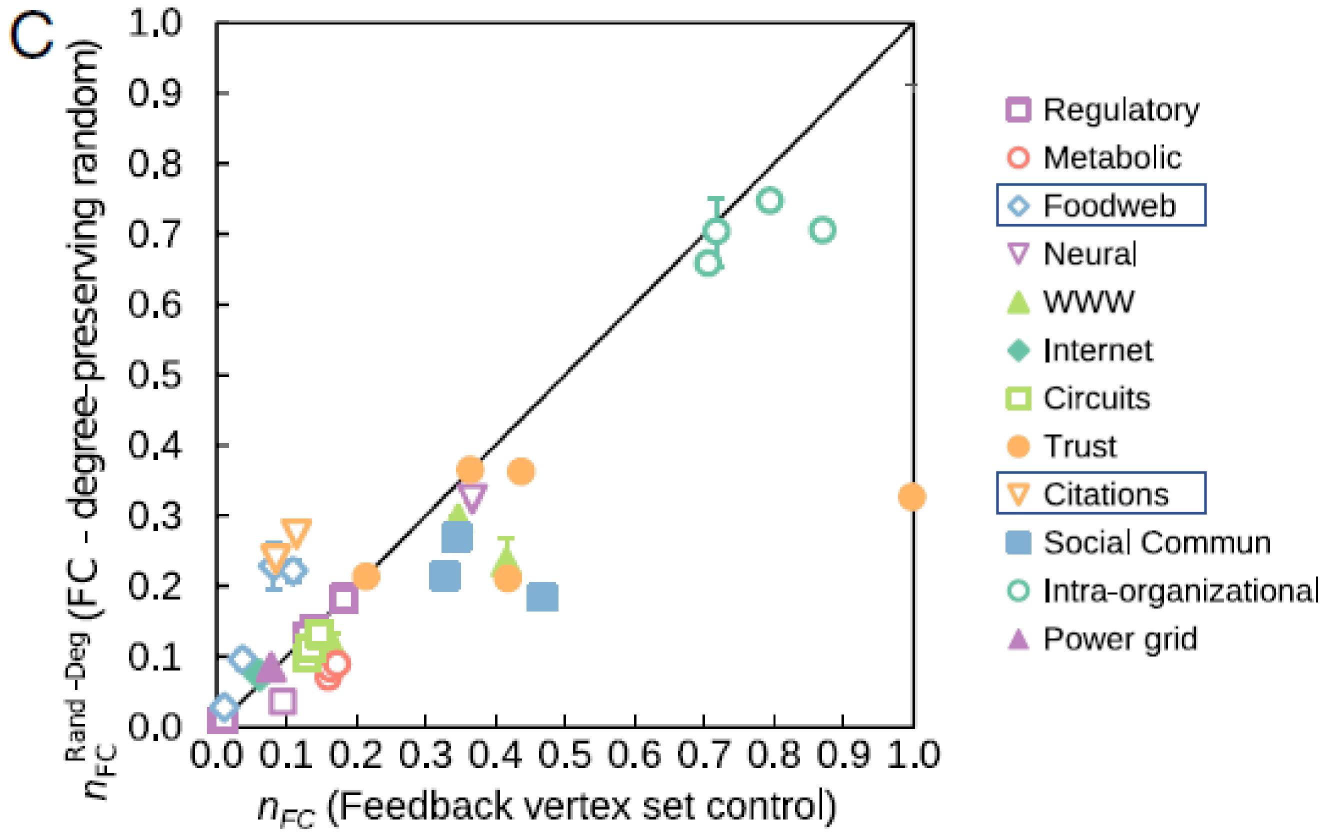
反馈节点集与强连通分量的联系



Every node that is involved in a cycle must also be part of a strongly connected component(SCC), a group of nodes in a network in which there is a directed path between any pair of nodes.

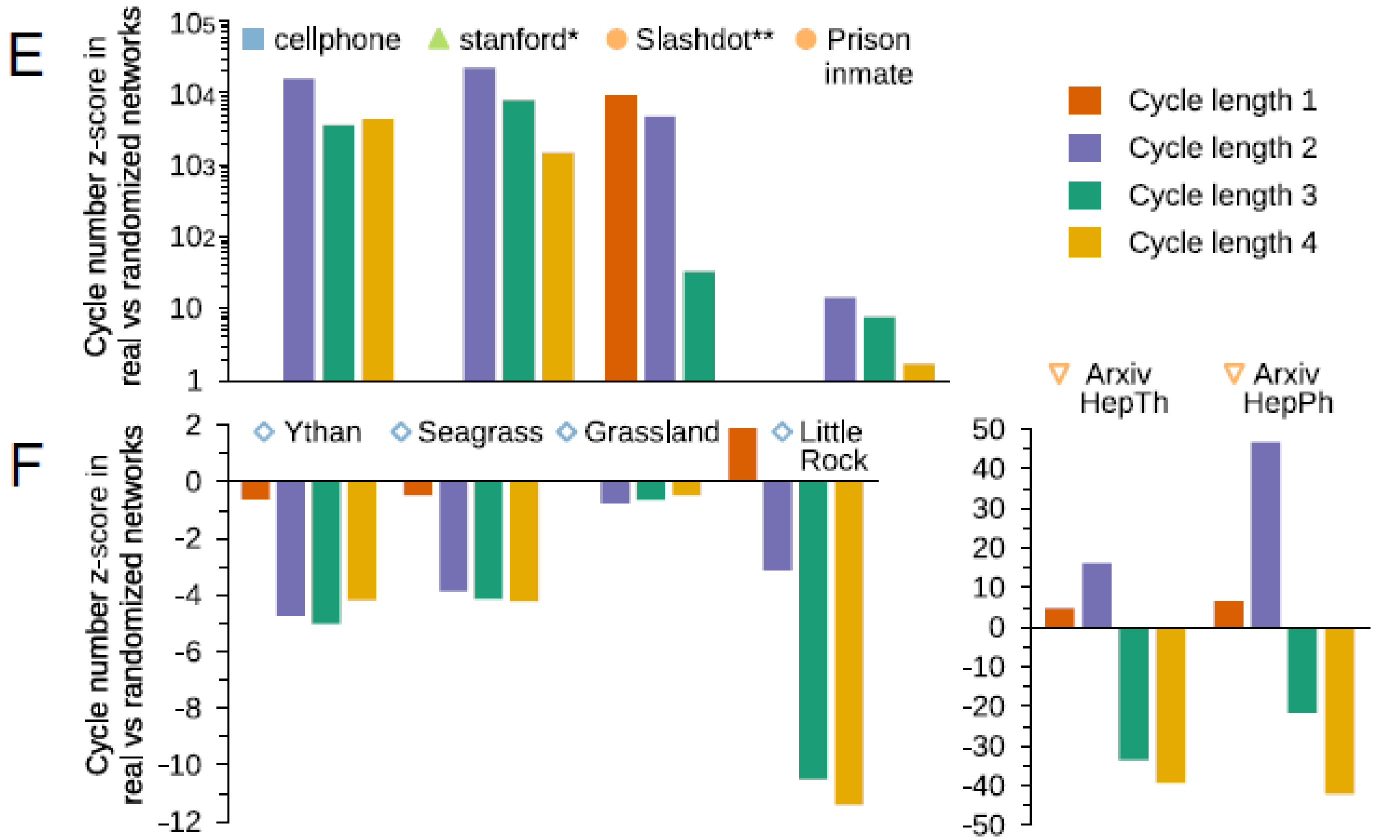
The networks show a strong correlation between the relative size of their SCCs (denoted by nSCC) and of their FVS

将FC理论应用于度保留随机网络



For most networks, the number of FC nodes is higher than the number of control nodes in randomize dversions ($n^{FC} > n_{FC}^{Rand-Deg}$), with the notable exceptions of food web and citation networks, in which randomized networks require more control nodes (n_{FC} $< n_{FC}^{Rand-Deg}$)

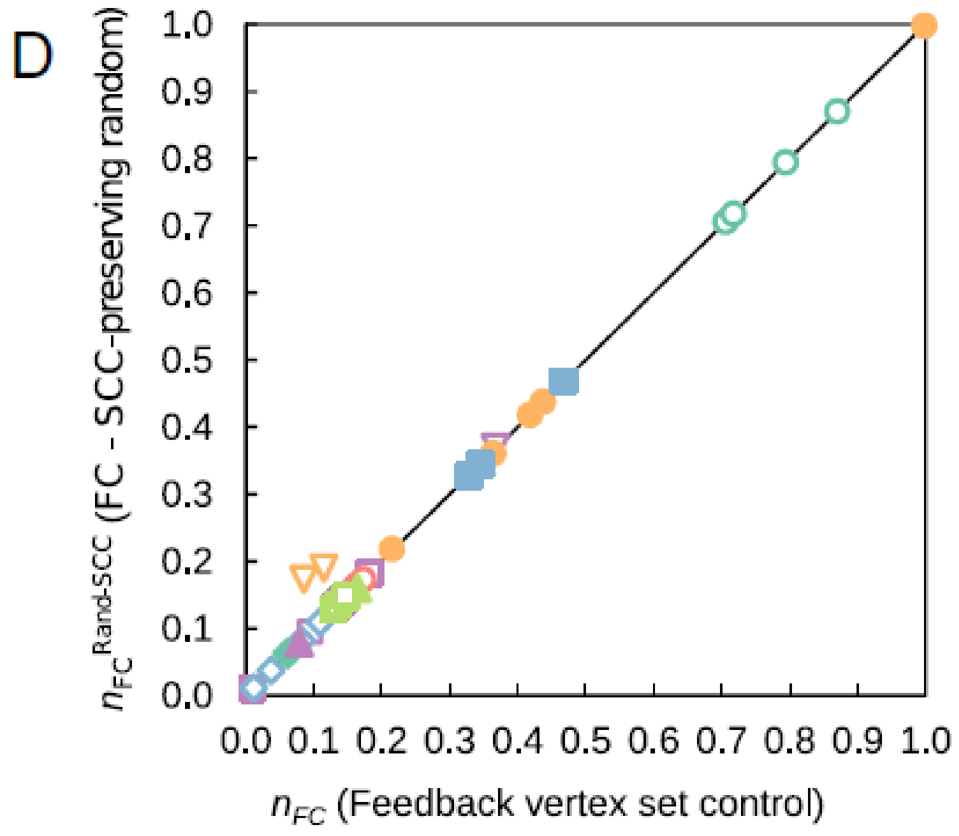
真实网络和随机网络循环结构长度与n_{FC}的关系



Real networks tend to have a more complicated cycle structure, evidenced by the overrepresentation of short cycles compared with the randomized networks, and reflected by the larger size of their FVS.

The exceptions to this reasoning are food web and citation networks, which are known to have an acyclic or close-to-acyclic structure , and thus feature fewer cycles and fewer nodes in an SCC than randomized networks.

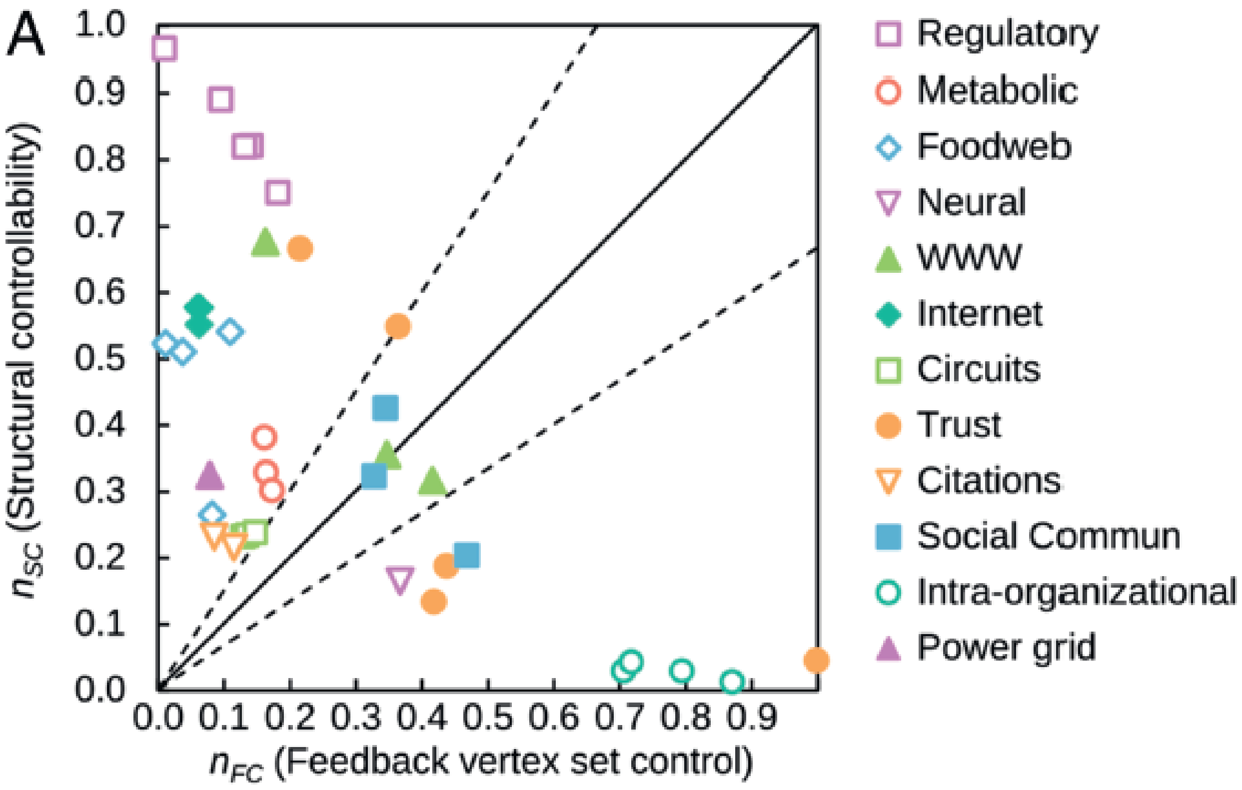
真实网络与强连通分量保留随机网络



To verify that the cycle structure of real networks explains the observed FC node set size, we generated degree-preserving randomized versions of these networks that maintain their cycle structure, which we achieve by randomizing the directed acyclic part of the graph while keeping intact the SCCs.

The cycle structure of a network, specifically its SCCs and short cycles, determines the number of nodes that need to be overridden in FC.

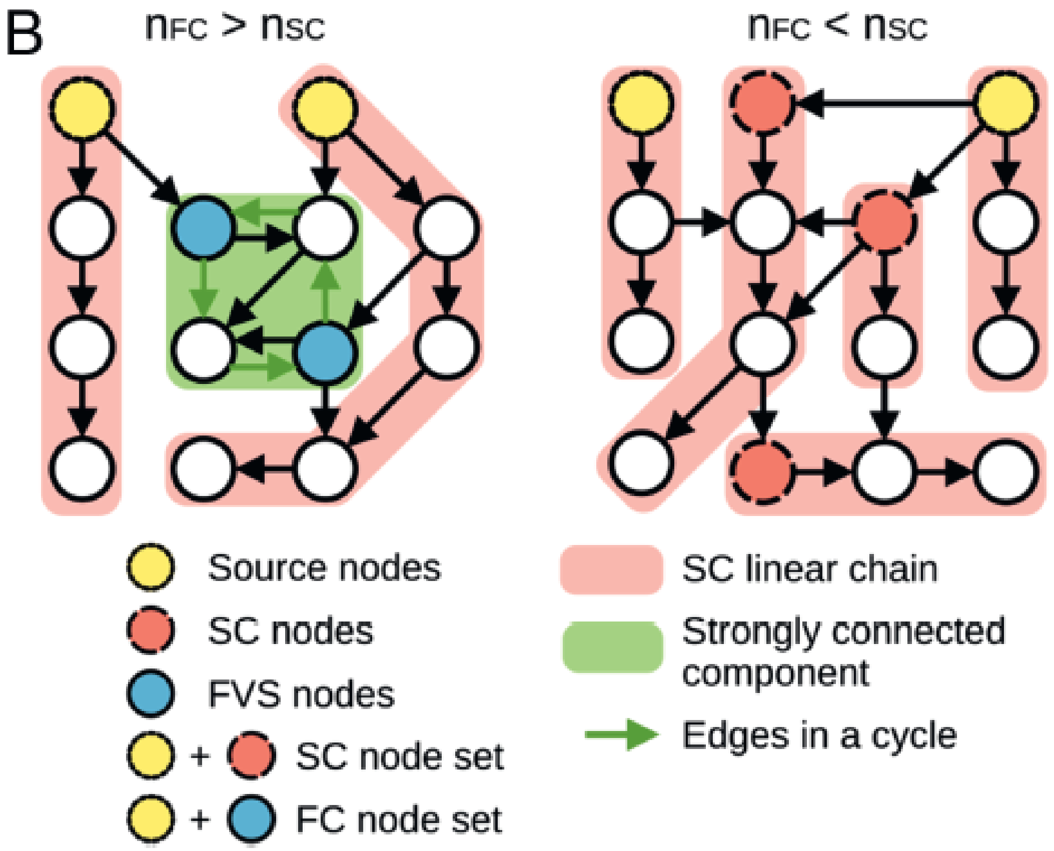
将反馈节点集控制理论与结构可控性理论比较



n_{SC} and n_{FC} appear to be inversely related across several types of networks.

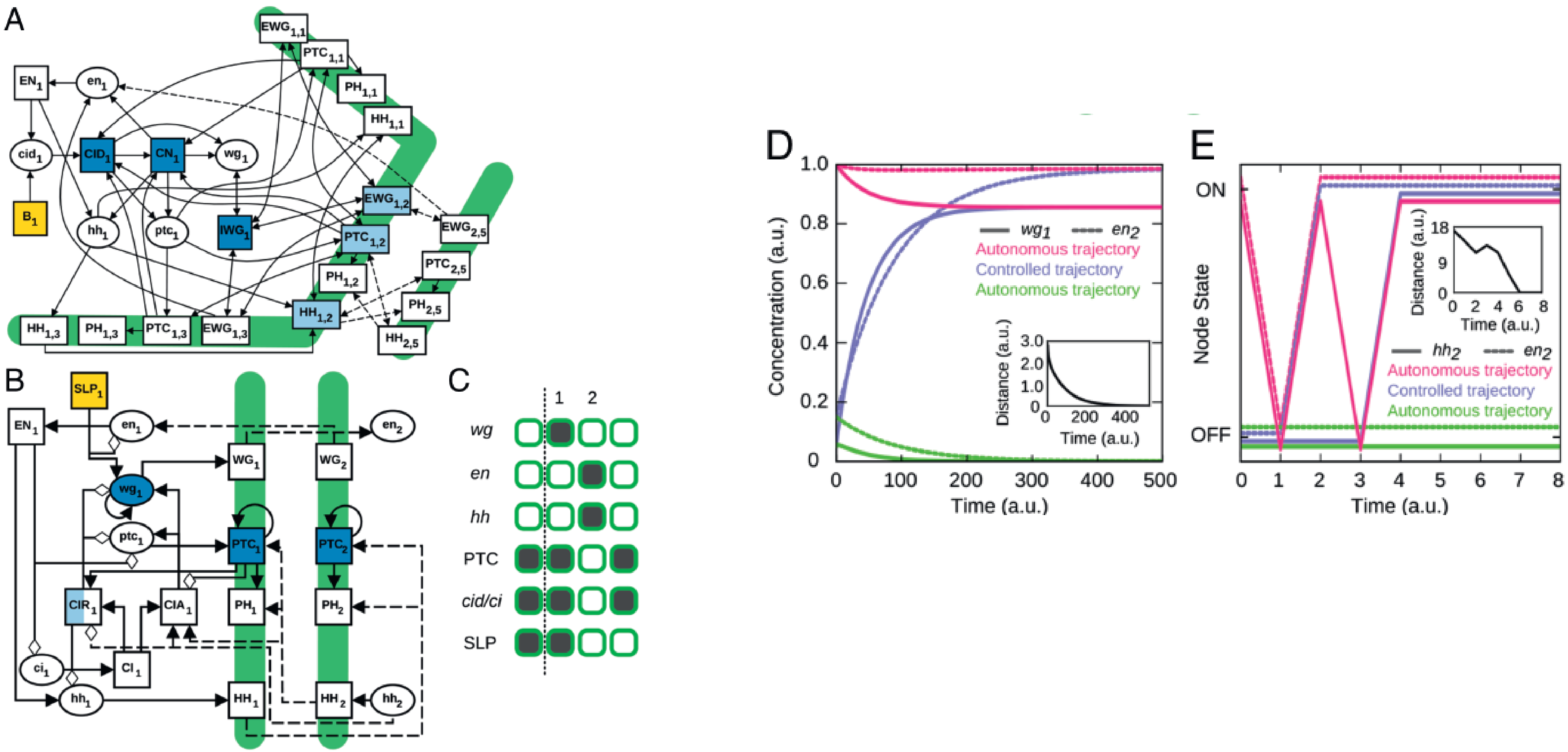
Cycles have to be controlled in FC but do not require independent control in SC.

进一步说明FC与SC的不同



SC additionally requires controlling the top nodes in other chains (red nodes with dashed outlines) but requires no independent control of cycles. FC requires controlling all cycles by control of the FVS (blue nodes with solid outlines).

在真实网络中的反馈节点集控制理论与动力学模型



讨论

- Attractor-based control (and thus FC) is the appropriate choice of control for biological systems, but also in many social and technological contexts.
- FC is directly applicable to systems in which only structural information is known, and also to systems in which a parameterized dynamic model is available, for which it provides realizable control strategies that are robust to changes in the parameters and functions.
- Further work is needed to extend FC and address topics such as the level of control provided by a subset of nodes, the task of building a controller signal that can implement the node state overrides, and the difficulty of steering the system toward a desired state, concepts that are well developed in control theory.

（1）用一句话概括此研究的主要结论和创新点

此研究验证了反馈节点集理论对复杂网络系统的广泛适用性，并比较了反馈节点集理论与结构可控性理论适用性的不同。为复杂网络的控制提供了新的思路。

（2）此研究对你有何启发

我们可以通过对复杂网络反馈节点集的控制来使系统达到达到理想的状态。

（3）此研究还存在哪些问题可以改进

我们对反馈节点集控制理论的缺陷以及具体适用性了解得并不详细，比如它具体适用于哪些类型的网络。